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FITTING OF THE TIDAL CONSTITUENTS TO THE CURRENT METER RECORDS REPRESENTING VELOCITY, PRESSURE AND THE CALCULATION OF SEMI-MAJOR AND SEMI-MINOR AXES OF THE TIDAL CURRENT ELLIPSES FOR M_2 CONSTITUENT

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Abstract. The tidal constituents O_1, K_1, M_2, S_2, M_4 and M_6 were fitted to the current meter records representing velocity and pressure at two stations in the Irish sea. Comparison of the amplitudes and phases of the *u* component of velocity (for dominant constituents M_2 and S_2) at different levels show that the phase of the tidal current was earlier at the bottom as expected and its amplitudes decreased from surface to bottom. The phase of the *v* component of velocity decreased, but the amplitude increased from surface to bottom. The calculated amplitudes and phases of the M_2 constituent for these positions are in reasonable agreement with the predicted values. The calculated semi-major and semi-minor axes are in fair agreement with the values deduced from Heaps model at position A but vary a lot at position B. This disagreement at position B may probably be due to the proximity to the coastal boundary of the model.

Introduction

The height of the tide at any time is represented by

$$h = H_0 + \Sigma f\ H \cos(\alpha t + (V_0+U) - K)$$

where h = height of the tide, H_0 = mean height of water level above datum, H = Mean amplitude of any constituent, f = Factor for reducing mean amplitude H to year of prediction α = angular speed of the constituent per unit time, normally in degrees per means solar day, t = time reckoned from some initial epoch, such as beginning of year of prediction, K = epoch of the constituent, (V_0+U) = Value of equilibrium argument of constituent when t = 0.

The initial epoch is usually taken at midnight at the beginning of the year. The quantity (V_0+U) is different for each constituent and is also different for each initial epoch and for different longitudes on the earth. The values of the quantity (V_0+U) have been compiled for the beginning of each year, each month of the year and each day of the month. To refer this Greenwich value of (V_0+U) to any local Meridian, it is necessary to apply a further correction equal to the product of the longitude L^o in degrees and the subscript P of the constituent which represents the number of periods in a constituent day. West longitude is to be considered as positive and east longitude as negative, and the subscript of the long period constituent are to be taken as zero. This correction is to be subtracted:

P = 1 when referring to diurnal constituents, P = 2 when referring to semi-diurnal constituents, P = 4 for quarter diurnal, P = 6 for sixth diurnal.

The phases for O_1, K_1, M_2, S_2 constituents were calculated from the Admiralty tide table part III[1] and they represent the high water time in angle after midnight for the respective constituent. The Manual of Harmonic analysis and prediction of tides[2] gives the phase of the constituent at 00.00 hr for each day. So for M_4 and M_6 these phases were subtracted from 360° to determine the high water angle after midnight. These phases were then divided by the speed (in degrees/hour) of the respective constituent. This quantity was called IP_i (i=1, 2---) for the different constituents. The speed of the constituents is given in Table 1.

Current Meter Data. The data were obtained from two current meter moorings (Fig. 1) deployed in 1975 at position A(53° 42'N, 3° 32.5'W) and position B (53° 35'N, 3° 22'W) in the Irish sea. Only position A was occupied in 1976. Table 2 shows the summary of the current meter deployment time and position. Computer programmes were developed to convert the recorded numbers into speed and direction. The values of the *u* component of velocity (positive towards East) and the *v* component of velocity (positive towards North) were obtained from speed and direction. Positive *v* component is in the true north direction on the x-y co-ordinate system. A low pass filter

$$\frac{\alpha^2_{12}\cdot\alpha_{14}}{12^2\cdot 14}$$

was used for smoothing and decimating the observations from a 5 min to a 1 hr sampling interval. Hourly residual values were obtained by the application of Godin's filter $\frac{\alpha^2_{24}\cdot\alpha_{25}}{24^2\cdot 25}$. This results in the loss of 35 data points on each end of the series. To overcome this, extrapolated values were generated by reflecting 35 values on each end of the series about the first (or last) data points. The hourly tidal values were then determined by subtracting the corresponding residual values from the smoothed hourly values, i.e. Hourly tidal values = Smoothed hourly values–Residual hourly values. i.e. Hourly tidal values = Smoothed hourly values = Residual values

TABLE 1. SPEED OF THE CONSTITUENTS IN DEGREES/DAY.

O_1	=	334.6340 degrees/day
K_1	=	360.9860 degrees/day
M_2	=	695.6200 degrees/day
S_2	=	720.0000 degrees/day
M_4	=	1391.2370 degrees/day
M_6	=	2086.8555 degrees/day

These 35 hourly residual values on either side, were in no way reliable and were omitted. However, the corresponding tidal values were considered reliable as the subtraction of a smaller number (residual values) from a bigger number (hourly values), did not introduce much error. This increased length of the record is an advantage in fitting the tidal constituents to the time series. Typical plots of the 5 min values, smoothed hourly values, hourly residual values and hourly tidal values of *u* component of the velocity for meter No. 556 are shown in Figs. 2a, 2b, 2c and 2d respectively.

Fitting of the Constituents O_1, K_1, M_2, S_2, M_4, M_6. The current meter record showed that the velocity varied predominantly with the tide. Much of the tidal energy is due to the semi-diurnal components of the tides. But the above six constituent were fitted to the hourly tidal values of the *u* and *v* components of velocity and pressure to calculate their respective phases and amplitudes according to the following equation:

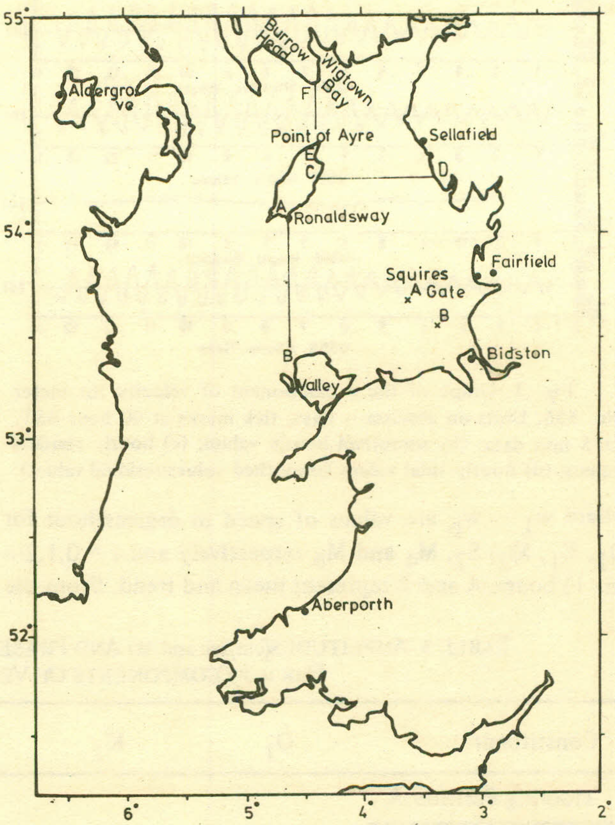

Fig. 1. Map of the Irish sea showing current meter mooring positions A and B in the Eastern Irish sea.

$$f(t)=A+Bt+C_1\cos w_1t+D_1\sin w_1t$$
$$+C_2\cos w_2t+D_2\sin w_2t$$
$$\vdots \qquad \vdots$$
$$+C_6\cos w_6t+D_6\sin w_6t$$

TABLE 2. SUMMARY OF THE CURRENT METER DEPLOYMENT TIME AND POSITION (Positions A and B).

Rig.	Latitude	Longitude	Water depth L-L-W	Day launched	Day recovered	Meter No. & type	Height of meter above sea floor	Recording interval
A	53° 42'	3° 32.5'	24M	31st July 1975	13th August 1975	556 (Plessey)	19M	5 min.
						626 (Plessey)	12.5M	5 min.
						629 (Plessey)	3M	5 min.
B	53° 35'	3° 22'	17M	31st July 1975	13th August 1975	406 (Bergen)	11M	5 min.
						236 (Bergen)	4M	5 min.
A	53° 42'	3° 32.5'	24M	30th June 1976	10th July 1976	1867 (Bergen)	19M	5 min.
						1750 (Bergen)	12.5M	5 min.
						1749 (Bergen)	3M	5 min.

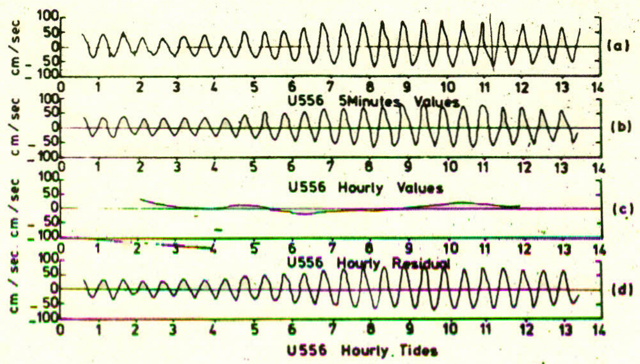

Fig. 2. Graph of the u component of velocity for meter No. 556. Units on abscissa – days, tick marks at 00 hour BST. (a) 5 min data; (b) smoothed hourly values; (c) hourly residual values; (d) hourly tidal values (smoothed values-residual values).

where $w_1 - w_6$ are values of speed in degrees/hour for O_1, K_1, M_2, S_2, M_4 and M_6 respectively and t = 0,1,2--- (n–1) hours. A and B represent mean and trend. From the values of C_i's and D_i's the amplitudes and phases of the constituents were calculated according to the following relations:

$$\text{Amplitude} = (C_i^2 + D_i^2)^{1/2}$$

$$\text{Phase} = \tan^{-1} \frac{(D_i)}{(C_i)}$$

If $\emptyset_i$ is the phase with respect to the local tide generating force constituent then:

$$\emptyset_i = (IT - IP_i) \times \text{speed in degrees/hour} + \tan^{-1} \frac{(D_i)}{(C_i)}$$

where IT is the time of the first sample of the series.

The amplitudes and phases of the six constituents (with respect to the local tide generating force constituent) calculated for the *u* and *v* components of velocity and pressure are shown in Table 3.

TABLE 3. AMPLITUDES(cm/sec and M) AND PHASES OF THE CONSTITUENTS O_1, K_1, M_2, S_2, M_4 and M_6 FOR *u*, *v* CCOMPONENTS OF VELOCITY AND PRESSURE (DEPTH).

Constituent	O_1	K_1	M_2	S_2	M_4	M_6
Mooring Position A						
(i) Top Current Meter						
(a) *u* Component of Velocity						
Amplitude (1975)	0.54	1.76	52.29	17.39	4.73	0.92
Amplitude (1976)	1.74	1.59	57.00	15.87	3.99	1.93
Phase (1975)	325.09	121.53	239.94	279.94	80.56	230.46
Phase (1976)	307.98	91.44	227.42	269.39	74.45	239.18
(b) *v* Component of Velocity						
Amplitude (1975)	0.64	0.29	8.80	4.61	3.22	0.63
Amplitude (1976)	0.79	0.39	9.95	1.41	2.22	0.32
Phase (1975)	108.63	0.36	359.69	79.35	220.78	82.20
Phase (1976)	206.30	56.96	356.64	345.22	204.96	317.37
(c) Pressure (depth)						
Amplitude (1976)	0.07	0.15	2.66	0.75	0.05	0.14
Phase (1976)	38.40	201.99	327.54	1.04	182.70	3.12
(ii) Middle Meter						
(a) *u* Component of velocity						
Amplitude (1975)	0.83	1.58	50.81	14.59	4.59	1.28
Amplitude 1976)	1.63	1.44	52.42	15.04	4.68	2.10
Phase (1975)	322.46	120.10	234.04	279.26	97.29	246.04
Phase (1976)	308.08	90.76	222.58	262.33	88.01	227.78

(Continued)

Table 3 (Continued)

(b) v Component of Velocity

Amplitude (1975)	0.43	0.33	11.94	6.19	1.88	1.04
Amplitude (1976)	0.55	0.54	15.65	3.00	1.13	0.44
Phase (1975)	88.82	78.54	349.65	32.08	199.49	107.10
Phase (1976)	204.03	79.30	345.31	25.28	145.85	355.20

(iii) Bottom Meter

(a) u Component of Velocity

Amplitude (1975)	0.70	1.12	42.86	17.68	4.26	0.89
Amplitude (1976)	1.37	1.13	45.05	13.27	3.57	1.39
Phase (1975)	216.14	96.93	218.09	265.32	111.98	214.90
Phase (1976)	297.62	83.26	218.22	256.55	76.68	222.19

(b) v Component of Velocity

Amplitude (1975)	0.52	0.29	18.53	5.88	0.73	0.28
Amplitude (1976)	0.57	0.31	17.23	3.15	1.26	1.02
Phase (1975)	78.08	97.88	344.41	22.13	218.84	98.22
Phase (1976)	232.14	100.45	329.11	8.65	339.72	287.06

(c) Pressure (depth)

Amplitude (1976)	0.05	0.15	2.61	0.72	0.08	0.09
Phase (1976)	17.32	205.70	323.42	358.26	180.08	9.49

Mooring Position B

(i) Top Current Meter

(a) u Component of Velocity

Amplitude 1975	0.73	0.58	52.97	17.84	5.68	2.05
Phase (1975)	312.42	138.82	238.48	281.45	71.65	259.93

(b) v Component of Velocity

Amplitude (1975)	0.85	0.54	9.84	5.18	0.50	0.64
Phase (1975)	153.24	350.53	60.08	107.53	177.53	154.23

(c) Pressure (depth)

Amplitude (1975)	0.10	0.12	2.63	0.88	0.13	0.14
Phase (1975)	42.58	203.17	329.67	10.75	188.56	18.21

(ii) Bottom Current Meter

(a) u Component of Velocity

Amplitude (1975)	0.68	1.18	37.39	14.66	3.57	1.95
Phase (1975)	324.40	139.34	233.18	282.91	92.44	210.01

(Continued)

Table 3 (Continued)

(b) v Component of Velocity						
Amplitude (1975)	.56	0.07	12.99	1.36	1.02	0.23
Phase (1975)	116.84	296.73	348.06	9.28	164.15	68.09

Comparison of Amplitudes and Phases of M_2 at Different Levels and with the Predicted Tidal Amplitude and Phases for that Position. Comparison of the amplitudes and phases for M_2 and S_2 of the u component of velocity at the level of top, middle and bottom current meters show that in both years the amplitude and phase decreased from surface to bottom. This was in agreement with the fact that the phase of the tidal current is earlier at the bottom and its amplitude and phase increase in going towards the surface. The phase of the v component of velocity decreased, but the amplitude increased from surface to bottom in both years. This increase in amplitude of the v component showed that the eccentricity of the tidal ellipses decreased from surface to bottom. This is also clear from the calculations of semi-major and semi-minor axes of the tidal ellipses.

The tidal amplitudes and phases of the M_2 elevations at positions A and B derived from the pressure records were compared with the predicted values for these locations. The predicted phases are with respect to Greenwich meridian and our calculated phases are with respect to local meridian, requiring a correction factor, of approximately 7 degrees (Product of p = 2 and L = 3° 30'). The predicted and the calculated amplitudes and phases are shown in Table 4. Comparison of the values show that they are in reasonable agreement at these positions.

Calculation of Semi-major and Semi-minor Axes for M_2 Constituent and Their Comparison with Those from Heaps Model. To determine the magnitude of the semi-major and semi-minor axes of the tidal current ellipses assume

$$u = a \cos(wt - g)$$

and $$v = b \cos(wt - g')$$

where a, b are the maximum amplitudes of the u, v components of velocity respectively.

g and g' are the respective phase lags.

Expanding $$u = a \cos wt \cos g + a \sin wt \sin g$$
$$v = b \cos wt \cos g' + b \sin wt \sin g'$$

Let $A = a \cos g \qquad B = a \sin g$
$A' = b \cos g' \qquad B' = b \sin g'$

we can write $u = A \cos wt + B \sin wt$
$v = A' \cos wt + B' \sin wt$

Defining an ellipse

$$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$$

with an angle α between u and u'

Let $-e$ be the time after t = 0 at which u' is a maximum, and $-e + \pi/2$ be the time after t = 0 at which v' is a maximum

we have $u' = a \cos(wt + e)$
$v' = b \sin(wt + e)$

we can also write

$$u' = a\ (\cos wt \cos e - \sin wt \sin e)$$
$$v' = b\ (\sin wt \cos e + \cos wt \sin e)$$

also

$$u' = u \cos\alpha + v \sin\alpha$$
$$v' = v \cos\alpha - u \sin\alpha$$

Solving for a and b

$$a = ((A \cos\alpha + A' \sin\alpha)^2 + (B \cos\alpha + B' \sin\alpha)^2)^{1/2}$$
$$b = ((A' \cos\alpha - A \sin\alpha)^2 + (B' \cos\alpha - B \sin\alpha)^2)^{1/2}$$

where $$\tan 2\alpha = \frac{2(AA' + BB')}{B^2 - (B')^2 + A^2 - (A')^2}$$

The calculated values of semi-major and semi-minor axes, together with those calculated by Heaps[3] on the basis of his two dimensional model, are shown in Table 5. The agreement between the values is very good at position A. However, at position B in 1975 the calculated values are about 100% higher than those from Heaps model. This

TABLE 4. COMPARISON OF THE CALCULATED AND PREDICTED AMPLITUDES AND PHASES (TIDAL ELEVATIONS) FOR POSITIONS A AND B.

Position	Year	Calculated		Predicted	
		Amplitude (M)	Phase (Degrees)	Amplitude (M)	Phase (degrees)
A (Top meter)	1976	2.66	327.54 + 7 = 334.54	2.9	318
B (Top meter)	1975	2.63	329.67 + 7 = 336.67	3.0	320

TABLE 5. COMPARISON OF THE CALCULATED SEMI-MAJOR AND SEMI-MINOR AXES WITH THOSE FROM THE HEAPS TWO DIMENSIONAL NUMERICAL MODEL.

Position		Calculated				From Heaps Model	
		Semi-major Axis (cm)		Semi-minor Axis (cm)		Semi-major Axis (cm)	Semi-minor Axis (cm)
		1975	1976	1975	1976		
A.	Top meter	52.48	57.35	7.61	7.66		
	Middle meter	51.08	53.14	10.71	12.99	52.4	8.3
	Bottom meter	44.42	45.53	14.41	15.93		
B.	Top meter	53.88		0.27		22.0	9.8
	Bottom meter	37.83		11.65			

was not unexpected because of the proximity to the coastal boundary of the model. In his model Heaps assumed straight coastal lines, which is not the case. The major axis decreased and minor axis increased from surface to bottom showing that the eccentricity of the tidal ellipses decreased from surface to bottom.

Conclusion

It can be concluded from the above analysis that:
(1) Most of the energy in the time series was due to the predominant M_2 and S_2 components. (2) The calculated amplitudes and phases of the M_2 tidal constituent are in reasonable agreement with the predictal values (Table 4). (3) The calculated semi-major and semi-minor axes are in fair agreement with those from Heaps model at position A (Table 5). However, at position B the values are 100% higher as expected. (4) The eccentricity of the tidal ellipses decreased from surface to bottom. (5) The phase of the tidal current was earlier at the bottom as expected and its amplitude decreased from surface to bottom (comparison of M_2 constituent at different levels).

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